Study on mitigating the systematic errors between the announced orthometric heights in different years by corrector surface models

Chia-Hsin Chen (1), Lao-Sheng Lin (1), Wei-Jun Wang (1), Yan-Yi Wu (1)

[1] Department of Land Economics, National Chengchi University
No. 64, Sec.2, Zhi-Nan Rd, Wenshan District, Taipei City 11605, Taiwan(R. O. C)
Email: 106257032@nccu.edu.tw; lslin@nccu.edu.tw;105207440@mail2.nccu.tw; amily115308@gmail.com

KEY WORDS: Systematic Error, Orthometric Height, Corrector Surface Model, GNSS Levelling

ABSTRACT: In Taiwan, the relative accuracies between benchmarks decreased due to the significant surface displacement with time and earthquake caused by the frequency activity of the crustal plate. Therefore, the Ministry of the Interior in Taiwan has re-surveyed the first-class benchmarks on the regular basis, and announced the orthometric heights of the benchmarks in different years, i.e. 2002, 2003, 2009 and 2015 respectively.
To mitigate the systematic errors between orthometric heights obtained in different time and then facilitate the implementation of GNSS leveling, various corrector surfaces model are proposed in this paper. According to the preliminary test results, they show that after applying the optimal corrector surface models on corresponding cites, the accuracies of the difference of orthometric heights in each city are significantly improved, which is helpful for improving the accuracy of GNSS leveling.

1. INTRODUCTION

The Global Positioning System (GPS) provides the observations with respect to geocentric World Geodetic System 1984 (WGS84). However, the so called ellipsoidal heights (h) derived by GPS must be transformed into orthometric heights (H) for practical applications (Cakir and Yilmaz, 2014). This transformation is applied with the knowledge of undulation (N) that must meet sufficient accuracy (Erol and Erol, 2013). Although orthometric heights, ellipsoidal heights and undulation are quite different, mainly in terms of physical meaning, datum definition, observational methods, they should satisfy the geometrical relationship as Eq. 1 shown (Kavzoglu and Saka, 2005).

$$\mathrm{H} = \mathrm{h} - \mathrm{N} \tag{1}$$

where H is the orthometric height, the distance of a point on the earth from the geoid along curved plumb line; h is the ellipsoidal height, the distance of a point on the earth from the surface of the reference ellipsoid along the normal; N denotes the undulation, the difference between ellipsoidal height and the orthometric height with respect to the geoid (Gullu et al., 2011).
According to Eq.1, it can be seen that if the undulation (N) of a point is known, then the ellipsoidal height of that points can be easily transformed to orthometric height. This process is referred to GPS leveling. Theoretically, orthometric height can be derived with the combination of GPS, e-GPS or e-GNSS observation and the undulation model with sufficient accuracy. Nevertheless, with frequent crustal plate movements in Taiwan, there is a decrease in relative accuracy between benchmarks. In other words, assume that the undulation hasn't changed, then orthometric heights on points are about to change with time.
The Ministry of the Interior in Taiwan announced the results of Taiwan's first-class leveling net in 2002, 2003, 2009 and 2015 respectively. If there are systematic errors exist among the orthometric heights announced in different years, they may be due to: (1) Taiwan is situated at the convergent boundary of the Philippine Sea and the Eurasian plates (Chen et al., 2011) (2) The variation in the orthometric heights announced in different years depend on time; (3) There are inherent inconsistencies exist among announced orthometric heights, etc (Lin, 2014). To mitigate the systematic errors, many corrector surface models like polynomial model, similarity transformation model, conicoid fitting method and artificial neural network have been proposed and fit the systematic errors well (Hu et al., 2002; Hu et al., 2004; Lin, 2014; Stopar et al., 2006). Therefore, for the purpose of fitting the orthometric heights announced in previous year to the latest one, corrector surface models mentioned above are used to fit the systematic errors between different orthometric heights announced in different years. Furthermore, the optimal corrector surface model are selected through some specific statistical tests.

2. METHODOLOGY

2.1 The difference of orthometric heights announced in different years

Assume that H^{92}, H^{98} and H^{104} represent the orthometric heights announced in 2003, 2009 and 2015 respectively. Then the difference among these three orthometric heights are expressed as Eq. (2) to Eq. (4):

$$\Delta H_i^{98-92} = H_i^{98} - H_i^{92} \tag{2}$$

$$\Delta H_i^{104-92} = H_i^{104} - H_i^{92} \tag{3}$$

$$\Delta H_i^{104-98} = H_i^{104} - H_i^{98} \tag{4}$$

$$\mathrm{i} = 1,2,\dots,\mathrm{n}$$

where n stands for the number of benchmarks.
In order to mitigate the systematic errors among the orthometric heights announced in different year, several corrector surface models are proposed in this study. An appropriate corrector surface model should absorb the inconsistencies of orthometric heights on points and allow the previous orthometric heights to fit the later one.
From Eq. (5), function $\mathrm{F}(x_i, y_i)$ or $a_i^T x$ represents the corrector surface model. This function can take various forms and complexity degree (Erol et al., 2008; Fotopoulos, 2003, Lin, 2014).

$$\Delta H_i = \mathrm{F}(x_i, y_i) + v_i = a_i^T x + v_i, i = 1,2,\dots,n \tag{5}$$

where ΔH_i means the orthometric height difference calculated by orthometric heights announced in different years; $\mathrm{x}(\mathrm{n} \times 1)$= vector of unknown parameters; $a_i(\mathrm{n} \times 1)$= vector of known coefficients, (x, y) represents the plane coordinates of points and v_i= residual term.

2.2 Corrector surface models

The corrector surfaces models used in this paper include polynomial model, similarity transformation model, conicoid fitting method and artificial neural network. Each method will be introduced respectively as following:

2.2.1 Polynomial model

The polynomial of degree n used to fit the systematic errors expressed as Eq. (6) (Cakir and Yilmaz, 2014):

$$\mathrm{F}(x_i, y_i) = \sum_{i=0}^{n} \sum_{j=0}^{n} a_{ij} x_i y_i \tag{6}$$

where (x_i, y_i) represents the plane coordinate of a point.; $\mathrm{F}(x_i, y_i)$ stands for the difference of orhometric heights announced in different years corresponding to (x_i, y_i); a_{ij} is the coefficient term of the polynomial. Four, six and ten parameters polynomial models expressed as Eq. (7) to Eq. (9) are used in this study.

$$\mathrm{F}(x_i, y_i) = a_0 + a_1 x_i + a_2 y_i + a_3 x_i y_i \tag{7}$$

$$\mathrm{F}(x_i, y_i) = a_0 + a_1 x_i + a_2 y_i + a_3 {x_i}^2 + a_4 x_i y_i + a_5 {y_i}^2 \tag{8}$$

$$\mathrm{F}(x_i, y_i) = a_0 + a_1 x_i + a_2 y_i + a_3 x_i y_i + a_4 {x_i}^2 + a_5 {y_i}^2 + a_6 {x_i}^3 + a_7 {y_i}^3 + a_8 {x_i}^2 y_i + a_9 x_i {y_i}^2 \tag{9}$$

2.2.2 Similarity transformation model

Eq. (10) represents the four-parameters similarity transformation model. From Eq. (10), vector x contains four elements. (Andritsanos et al., 2000; El-Mowafy et al., 2006; Fotopoulos, 2003; Iliffe et al., 2003; Kotsakis and Katsambalos, 2010; Lin, 2014; Vella, 2003; Ziebart et al., 2004)

$$\mathrm{F}(x_i, y_i) = x_1 + x_2 cos\varphi_i cos\lambda_i + x_3 cos\varphi_i sin\lambda_i + x_4 sin\varphi_i \tag{10}$$

The five-parameters and seven-parameters similarity transformation model shown in Eq. (11) and Eq. (12) are expanded by Eq. (10) (Abdalla and Fairhead, 2011; Benahmed Daho, 2010; Fotopoulos, 2003; Kiamehr, 2011; Vella, 2003):

$$\mathrm{F}(x_i, y_i) = x_1 + x_2 cos\varphi_i cos\lambda_i + x_3 cos\varphi_i sin\lambda_i + x_4 sin\varphi_i + x_5 sin^2\varphi_i \tag{11}$$

$$\mathrm{F}(x_i, y_i) = x_1 \; cos\varphi_i cos\lambda_i + x_2 cos\varphi_i sin\lambda_i + x_3 sin\varphi_i + x_4 \left(\frac{sin\varphi_i cos\varphi_i sin\lambda_i}{W}\right) + x_5 \left(\frac{sin\varphi_i cos\varphi_i cos\lambda_i}{W}\right) + x_6 \left(\frac{1-f^2 sin^2\varphi_i}{W}\right) + x_7 \left(\frac{sin^2\varphi_i}{W}\right) \tag{12}$$

where $\mathrm{w} = \sqrt{1 - \mathrm{e}^2 \sin^2 \varphi_i}$; $\mathrm{e}^2 =$ eccentricity; f = flattening of the reference ellipsoid.

2.2.3 Conicoid fitting method

The conicoid fitting method is usually used to establish the undulation model using geometric method. (Hu et al., 2004; Lin, 2007). Three types of conicoid fitting method are tested in this study, which are four-parameters, six-parameters and ten-parameters as Eq. (13), Eq. (14) and Eq. (15) expressed respectively.

$$\mathrm{F}(x_i, y_i) = x_1 + x_2\varphi_i + x_3\lambda_i + x_4\varphi_i\lambda_i \tag{13}$$

$$\mathrm{F}(x_i, y_i) = x_1 + x_2\varphi_i + x_3\lambda_i + x_4\varphi_i\lambda_i + x_5\varphi_i^2 + x_6\lambda_i^2 \tag{14}$$

$$\mathrm{F}(x_i, y_i) = x_1 + x_2\varphi_i + x_3\lambda_i + x_4\varphi_i\lambda_i + x_5\varphi_i^2 + x_6\lambda_i^2 + x_7\varphi_i^3 + x_8\varphi_i^2\lambda_i + x_9\varphi_i\lambda_i^2 + x_{10}\lambda_i^3 \tag{15}$$

Suppose that there are n benchmarks with its known plane coordinates and corresponding $\Delta\mathrm{H}$ distributed in a specific test area, then any corrector surface model mentioned above can be used to fit $\Delta\mathrm{H}$. The matrix form of observation function can be expressed as follows (Lin, 2014):

$$\mathrm{Ax} = \Delta\mathrm{H} + \mathrm{v} \tag{16}$$

where A = design matrix composed of a_i^T for each $\Delta\mathrm{H}_i$. The unknown parameters x can be determined through least-squares adjustment (Ghilani, 2010).

2.2.4 Artificial neural network

Neural networks (also known as artificial neural network) were composed of artificial neurons. The behavior of artificial neurons is based on the decision-making process of a human brain. The input information of the neuron is multiplied by the synaptic weights adjusted during a training process, and then they are added and subjected to an activation function that generates the output information. (Lin, 2014; Veronez et al., 2006; Gullu et al., 2011).

Back-propagation (BP) ANN is a multilayer feed-forward network and a supervised learning network. Feed-forward networks often have one input layer and one or more hidden layers of sigmoid neurons followed by an output layer of linear neurons (Hu et al., 2004; Kavzoglu and Saka, 2005; Lin, 2007; Lin, 2014). In this paper, a three-layer BP ANN with one input layer, one hidden layer, and one output layer was adopted to generate a corrector surface model It is complicated when applying BP ANNs. Especially for the specification of the number and size of the hidden layer(s) and the choice of proper values for network parameters, they would affect the network's learning ability, generalization, and the performance of the learning algorithm. Therefore, it is often the case that a number of experiments are required to ascertain the selection of the parameter values that give the highest accuracy. On the other hand, A trial-and-error strategy is frequently used to determine appropriate values for these parameters (Hu et al., 2004; Kavzoglu and Saka, 2005; Lin, 2007; Lin, 2014).

Suppose there are n reference points in a specific region. The reference point set $\mathrm{P} = (P_1, P_2, \ldots, P_n)$ shown as Eq. (17) are used to train the BP ANN.

$$P_i = (x_i, y_i, \Delta H_i), i = 1,2, \ldots, n \tag{17}$$

From Eq. (17), the input vector consists of points' plane coordinate (x_i, y_i) and the output vector is composed of corresponding difference of orthometric heights ΔH_i. On the other hand, the number of neuron in hidden layer is determined by trail and error.

After the training of data set, the function between input layer (x_i, y_i) and output layer ΔH_i is shown as Eq. (18).

$$\Delta H_i = F(x_i, y_i),\ i = 1,2, \dots n \tag{18}$$

where $F(x_i, y_i)$= function that associates input vectors (x_i, y_i) with specific output vectors ΔH_i. It should be noted that $F(x_i, y_i)$ is similar to the coefficients of the parametric models. However, the function of $F(x_i, y_i)$ is determined implicitly by the neurons in the hidden layer of the BP ANN (Lin, 2014).

2.3 Performance evaluation and statistical analysis procedures

To mitigate the systematic errors of the orthometric heights announced in different years, various corrector surface models were applied . In this way, the optimal corrector surface model can be determined and then the orthomatric heights announced in previous year can be fit to the later one. All the test data were separated into two groups: reference points and check points. The reference points were used to estimate the coefficients of parametric models or train BPANN. The check points were used to estimate the performance of corrector surface models.

The optimal corrector surface model was determined based on a series of statistical tests. First, t-test was conducted to examine whether the systematic errors of orthometric heights announced in different years exists or not. Then various methods (ex: similarity transformation) mentioned above were applied for the area containing systematic errors. Afterwards, three optimal corrector surface model candidates: optimal, sub-optimal, and the third best corrector surface model can be determined.

With these three corrector surface model being applied on the primary orthometric heights, t-test and χ^2-test was conducted to determine the optimal corrector surface model of the specific test area.

In summary, three types of statistical tests and calculation methods were further implemented to estimate the performance of three optimal corrector surface model candidates: (1) the improvement in σ; (2) a two-tailed t-test on the mean value of ΔH; (3) a two-tailed χ^2-test on the variance of ΔH.

The improvement in σ were defined as Eq. (19) shown.

$$\text{The improvement in } \sigma = \frac{\sigma_{prefit} - \sigma_{postfit}}{\sigma_{prefit}} \times 100\% \tag{19}$$

where σ_{prefit} and $\sigma_{postfit}$ represent the standard deviation of ΔH at the check points before and after applying the specific corrector surface model.

A two-tailed t-test with a significance level of α =5% was conducted to examine if there is any systematic errors occurred in the sample as a whole by testing the deviation of the sample mean from the mean of its population (assumed to be zero). This test involves checking the sample mean ($\bar{y}$) of ΔH from the check points against the population mean ($\mu = 0.000\text{m}$). Moreover, the null hypothesis in this test is $H_0: \mu = \bar{y}$; on the other hand, the alternative hypothesis is $H_a: \mu \neq \bar{y}$ (Ghilani 2010).

A two-tailed χ^2-test with a significance level of α =5% was performed to check : (1) if the variance of ΔH at the check points (after applying a optimal corrector surface model) was the same as after applying an sub-optimal and the third best corrector surface model. (2) if the variance of ΔH at the check points (after applying a optimal corrector surface model) was the same as before applying any corrector surface model. The test involved: (1) checking the variance of ΔH(S^2) after applying specific optimal corrector surface model against the variance of ΔH(σ^2) after applying an sub-optimal and the third best corrector surface model. (2) checking the variance of ΔH(S^2) after applying specific optimal corrector surface model against the variance of ΔH(σ^2) before applying any corrector surface model. Furthermore, the null hypothesis in this test is $H_0: S^2 = \sigma^2$; on the other hand, the alternative hypothesis is $H_a: S^2 \neq \sigma^2$ (Ghilani 2010).

3. STUDY AREA AND TEST DATA

The study area selected in this paper is Taiwan region. The test data distributed in study area were first-class benchmaks with known plane coordinates and orthometric heights published in 2002, 2003, 2009 and 2015 respectively. Considering the surface relief caused by natural and human factors, the study area was divided into five segments: north area, middle area, south area, east area and land-subsidence area. As Fig.1 shown, the north area contains Keelung city, Taipei city, New Taipei city, Taoyuan city and Hsinchu county/city. The middle area contains Miaoli county, Taichung city and Nantou county. The land-subsidence area contains Changhua county, Yunlin county, Chiayi county/city and Tainan city. The south area contains Kaohsiung city and Pingtung county. The east area contains Yilan county, Hualian county and Taitung city. The number of points distributed in every county/city were listed in Table 1.

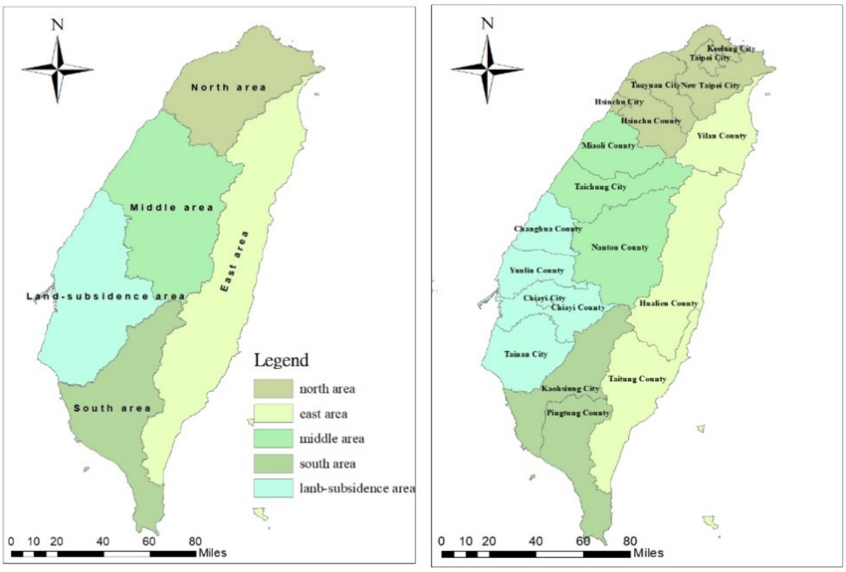

Fig. 1 Five parts of the test area (left graph) and its corresponding cities/counties included (right graph)

Table 1. The number of points distributed in every county/city

North area	Keelung	New Taipei	Taipei	Taoyuan	Hsinchu
Number	13	91	27	78	63
Middle area	Miaoli	Taichung	Nantou		
Number	55	96	73		
South area	Kaohsiung	Pingtung			
Number	135	146			
East area	Yilan	Hualian	Taitung		
Number	117	132	102		
Land-subsidence area	Changhua	Yunlin	Chiayi	Tainan	
Number	73	28	90	118	

4. DATA ANALYSIS

4.1 The variation in ΔH

Due to the limited page, only three cities/counties were represented their variation in ΔH in this paper as Fig. 2 to Fig. 4 shown. From Fig. 2 to Fig. 4, it can be seen that the variation in ΔH in each city/county is different with one another. On the other hand, the results of t-test conducted to examine whether the systematic errors in ΔH exist or not were shown in Table 2. In Table 2, 「T1」,「T2」 and 「T3」 represents ΔH_{98-92}, ΔH_{104-98} and ΔH_{104-92} respectively;「Ⓨ」 means the object contains systematic errors. From Table 2, it can be seen that most cities/counties contain systematic errors.

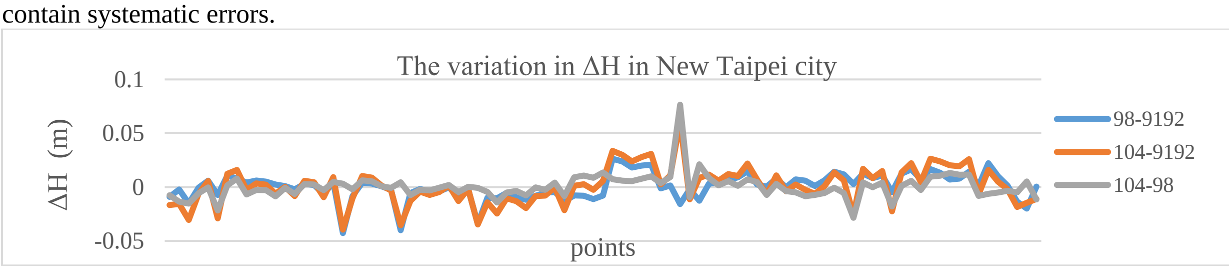

Fig. 2 The variation in $\Delta H_{98-92}, \Delta H_{104-98}$ and ΔH_{104-92} in New Taipei city.

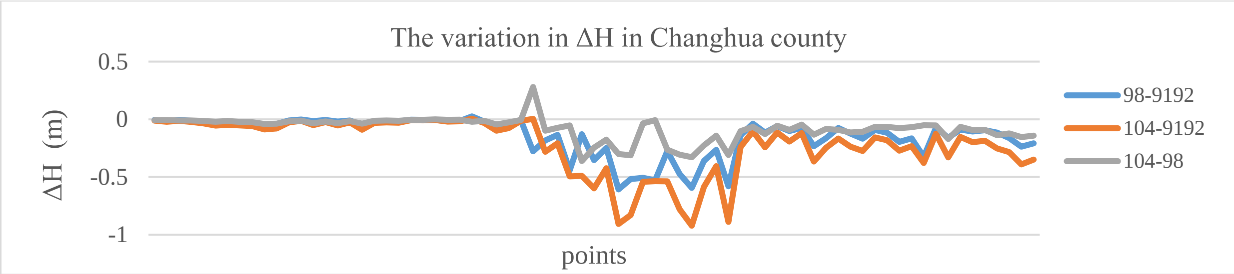

Fig. 3 The variation in $\Delta H_{98-92}, \Delta H_{104-98}$ and ΔH_{104-92} in Changhua county.

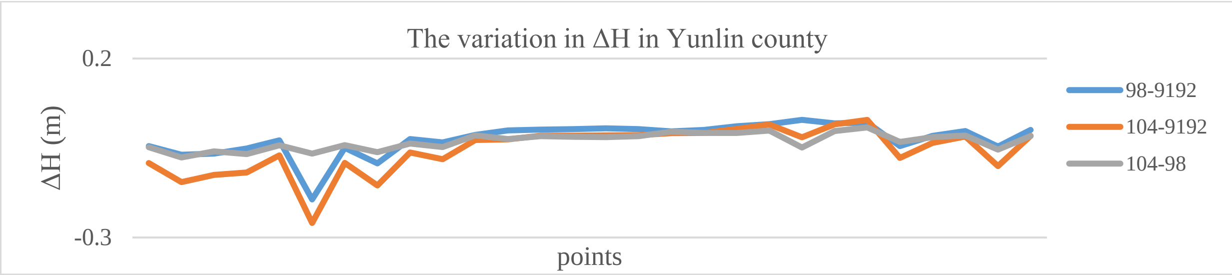

Fig. 4 The variation in $\Delta H_{98-92}, \Delta H_{104-98}$ and ΔH_{104-92} in Yunlin county.

Table. 2 The results of conducting t-test on mean value of $\Delta H_{98-92}, \Delta H_{104-98}$ and ΔH_{104-92} in each city/county 「T1」,「T2」and 「T3」 represents ΔH_{98-92}, ΔH_{104-98} and ΔH_{104-92} respectively;「Ⓨ」means the object contains systematic errors

		T1	T2	T3
North	Keelung			
	New Taipei			
	Taipei	Ⓨ		Ⓨ
	Taoyuan	Ⓨ	Ⓨ	Ⓨ
	Hsinchu	Ⓨ		Ⓨ
Middle	Miaoli	Ⓨ	Ⓨ	
	Taichung	Ⓨ	Ⓨ	Ⓨ
	Nantou	Ⓨ	Ⓨ	Ⓨ
South	Kaohsiung	Ⓨ	Ⓨ	Ⓨ
	Pingtung	Ⓨ		
East	Yilan		Ⓨ	
	Hualian			Ⓨ
	Taitung	Ⓨ	Ⓨ	Ⓨ
Land-subsidence	Changhua	Ⓨ	Ⓨ	Ⓨ
	Yunlin	Ⓨ	Ⓨ	Ⓨ
	Chiayi	Ⓨ	Ⓨ	Ⓨ
	Tainan			

4.2 Corrector surface models establishments and systematic errors mitigation

To mitigate the systematic errors in ΔH in each city/county, two main solutions were used in this paper. One is using single corrector surface model to fit the systematic errors in ΔH for each city/county (e.g. Taipei city shown in Table 3). As for the cities/counties whose systematic errors ΔH cannot be mitigate and allow the standard deviation of ΔH improved significantly with single corrector surface model, the test data distributed in that cities/counties were then separated into two groups (e.g. area A and area B of Changhua shown in Fig. 5) to establish the respective corrector surface models (e.g. Changhua county/city shown in Fig. 5 and Table 4).

In Table 3 and Table 4, the optimal corrector surface model denotes the tested corrector surface model has the smallest standard deviation of ΔH in each determined model. The σ_{prefit} and $Mean_{prefit}$ represents the standard deviation and the mean value of ΔH before applying corrector surface model respectively. The $\sigma_{postfit}$ and $Mean_{postfit}$ represents the standard deviation and the mean value of ΔH after applying corrector surface model respectively. According to Table 3 and Table 4, it can be seen that: (1) the optimal corrector surface model of ΔH_{98-92}, ΔH_{104-92} in Taipei city and ΔH_{98-92}, ΔH_{104-98} and ΔH_{104-92} in Changhua county are $2 \times 35 \times 1$ BPANN, six-degree polynomial, 4-parameter similarity transformation + 5-parameter similarity transformation, 4-parameter conicoid fitting + 10-parameter conicoid fitting and 4-parameter conicoid fitting + 10-parameter conicoid fitting respectively; (2) the standard deviation of ΔH significantly reduced and the mean values of ΔH more approximate to zero after applying an optimal corrector surface model.

Table 3. Performance of ΔH in Taipei city before and after applying the optimal corrector surface model

City/ County		Optimal corrector surface model	σ_{prefit} (m)	$\sigma_{postfit}$ (m)	$Mean_{prefit}$ (m)	$Mean_{postfit}$ (m)
Taipei	ΔH_{98-92}	$2 \times 35 \times 1$ BPANN	0.0070	0.0036	0.0067	-0.0013
	ΔH_{104-92}	Six-degree polynomial	0.0100	0.0044	0.0056	-0.0007

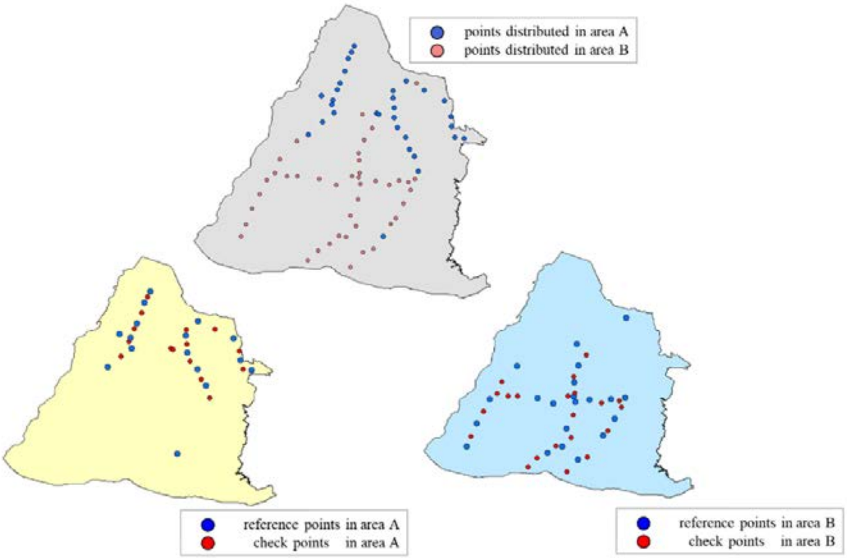

Fig 5. The points distribution in area A and area B of Changhua (the upper figure) and their reference points and check points distribution (the bottom figure)

Table 4. Performance of ΔH in Changhua before and after applying the optimal corrector surface model

City/ time/area			Optimal corrector surface model	σ_{prefit} (m)	$\sigma_{postfit}$ (m)	$Mean_{prefit}$ (m)	$Mean_{postfit}$ (m)
Changhua	ΔH_{98-92}	A	4-parameter similarity transformation	0.1656	0.0393	0.1434	-0.0016
		B	5-parameter similarity transformation				
	ΔH_{104-98}	A	4-parameter conicoid fitting	0.1014	0.0346	0.0800	0.0029
		B	10-parameter conicoid fitting				
	ΔH_{104-92}	A	4-parameter conicoid fitting	0.2415	0.0444	0.2235	0.0032
		B	10-parameter conicoid fitting				

4.3 The standard deviation of ΔH before and after applying the specific corrector surface model for each county/city

Fig. 6 to Fig. 8 show the variation of the standard deviation of ΔH before and after applying the specific corrector surface model for each county/city. It can be seen that the standard deviation of ΔH in each listed city/county was reduced. On the other hand, from the results of t-test($\alpha = 5\%$), there are no systematic errors in ΔH after applying the specific corrector surface models. Moreover, according to the results of χ^2-test($\alpha = 5\%$), the standard deviation of ΔH reduced significantly after applying the specific corrector surface models.

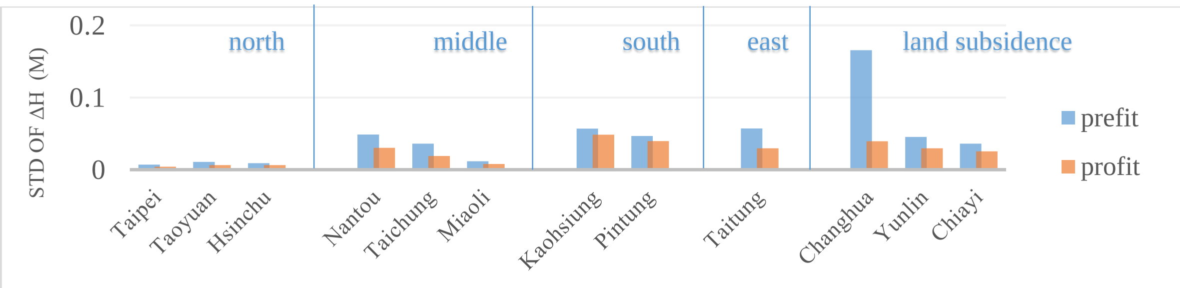

Fig 6. The standard deviation of ΔH_{98-92} before and after applying the specific corrector surface models

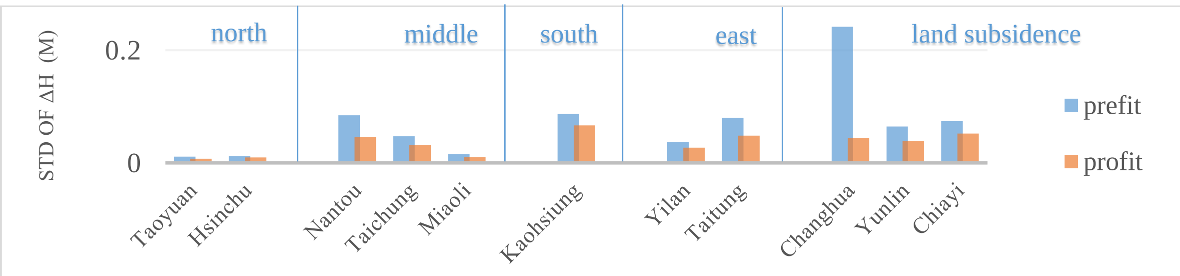

Fig 7. The standard deviation of ΔH_{104-98} before and after applying the specific corrector surface models

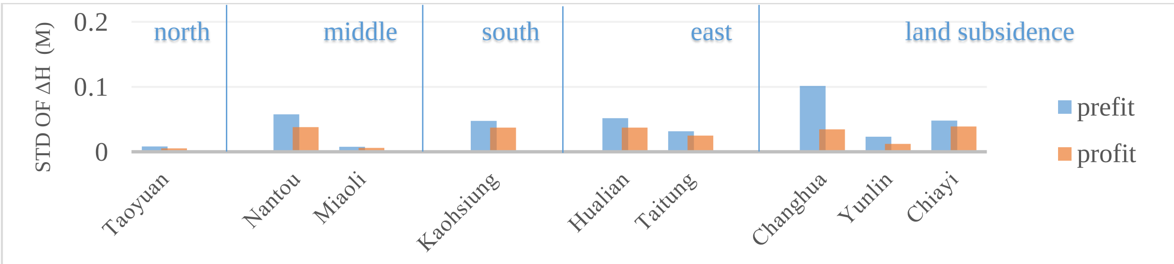

Fig 8. The standard deviation of ΔH_{104-92} before and after applying the specific corrector surface models

5. CONCLUSIONS

To discuss the variation among the ΔH calculated by orthometric published in different year, and to mitigate the systematic errors contain in ΔH for the purpose of improving the accuracy of GPS/GNSS leveling, the test data established in 2002, 2003, 2009 and 2015 are used in this paper.

With the complexity of surface relief, various mathematical fitting methods (e.g. the conicoid fitting model, the similarity transformation model and the polynomial model) proposed in this paper may have their limitation owing to their model limitation Therefore, other algorithm can be applied for the application with high accuracy requirement.

The issues discussed in this paper includes: (1) Analyze and examine the variation in ΔH; (2) T-tests was used to examine whether the systematic errors of ΔH exists or not. (3) The corrector surface models were established for the area containing systematic errors. (4) χ^2-test was used to examine whether the standard deviation of ΔH improved after the specific corrector surface model was applied, etc.

According to the test result, it can be seen that: (1) Due to the different variation in ΔH in each region, the corrector surface models varies from place to place. (2) There are no systematic errors exist after the proper corrector surface models were applied. (3) The accuracies of the difference of orthometric heights in each city are improved significantly , which is helpful for improving the accuracy of GNSS leveling.

6. REFERENCE

1. Abdalla, A., and Fairhead, D., 2011. A new gravimetric geoid model for Sudan using the KTH method. *J. Afr. Earth Sci.*, Vol. 60, No.4, pp. 213–221.
2. Andritsanos, V. D., Fotiou, A., Paschalaki, E., Pikridas, C., Rossikopoulos, D., and Tziavos, L. N., 2000. Local geoid computation and evaluation. *Phys. Chem. Earth*, Vol. 25, No. 1, pp. 63–69.
3. Benahmed Daho, S. A., 2010. Precision assessment of the orthometric heights determination in northern part of Algeria by combining the GPS data and the local geoid model. *C. R. Geosci.*, Vol. 342, No.2, pp. 87–94.
4. Cakir, L., and Yilmaz, N., 2014. Polynomials, radial basis functions and multilayer perceptron neural network

methods in local geoid determination with GPS/levelling. *Measurement*, Vol. 57, pp. 148-153.
5. Chen, C. H., Yang, M., and Tseng, C. L., 2004. Improving local geoid model of Taiwan using TWVD2001 leveling and GPS data: With application to GPS heighting. *Journal of Surveying Engineering*, Vol. 46, No. 1, pp. 1-20 (in Chinese).
6. El-Mowafy, A., Fashir, H., Al Habbai, A., Al Marzooqi, Y., and Babiker, T., 2006. Real-time determination of orthometric heights accurate to the centimeter level using a single GPS receiver: Case study. *J. Surv. Eng*., Vol. 132, No. 1, pp. 1–6.
7. Erol, B., Erol, S., and Celik, R. N., 2008. Height transformation using regional geoids and GPS/leveling in Turkey. Surv. Rev., Vol. 40, No. 307, pp. 2–18.
8. Erol, B., and Erol, S., 2013. Learning-based computing techniques in geoid modeling for precise height transformation. Computers & geosciences, Vol. 52, pp. 95-107.
9. Fotopoulos, G., Kotsakis, C., and Sideris, M.G., 2003. How accurately can we determine orthometric height difference from GPS and geoid data?, Journal of Surveying Engineering, Vol. 129, No. 1, pp. 1-10.
10. Ghilani, C. D., 2010. Adjustment Computations: Spatial Data Analysis, 5th Edition, Wiley, John Wiley & Sons, Inc.
11. Gullu, M., Yilmaz, M., and Yilmaz, I., 2011. Application of back propagation artificial neural network for modelling local GPS/levelling geoid undulations: A comparative study. In FIG Working Week, pp. 18-22.
12. Hu, W., Kong, Y., Deng, Y., and Kuang, S., 2002. A new method to derive normal height from GPS height based on neural network. *FIG XXII International Congress*, Washington, D.C. USA, April 19-26, pp. 1-11.
13. Hu, W., Sha, Y., and Kuang, S., 2004. New method for transforming global positioning system height into normal height based on neural network. *Journal of Surveying Engineering*, Vol. 130, No. 1, pp. 36-39.
14. Iliffe, J. C., Ziebart, M., Cross, P. A., Forsberg, R., Strykowski, G., and Tscherning, C. C., 2003. OSGM02: A new model for converting GPS-derived heights to local height datums in Great Britain and Ireland. *Surv. Rev.*, Vol., 37, No. 290, pp. 276–293.
15. Kavzoglu, T., and Saka, M. H., 2005. Modelling local GPS/levelling geoid undulations using artificial neural networks. Journal of Geodesy, Vol. 78, No. 9, pp. 520-527.
16. Kotsakis, C., and Katsambalos, K., 2010. Quality analysis of global geopotential models at 1542 GPS/leveling benchmarks over the Hellenic Mainland. *Surv. Rev*., Vol. 42, No. 318, pp. 327–344.
17. Kiamehr, R., 2011. The new quasi-geoid model IRQ09 for Iran. *J. Appl. Geophys*., Vol., 73, No. 1, pp. 65–73.
18. Lin, L.S., 2007. Application of a Back-Propagation Artificial Neural Network to Regional Grid-Based Geoid Model Generation Using GPS and Leveling Data, *Journal of Surveying Engineering*, Vol. 133, No. 2, pp. 81-89.
19. Lin, L.S., 2014. Orthometric Height Improvement in Tainan City using RTK GPS and Local Geoid Corrector Surface Models, *Journal of Surveying Engineering*, Vol.140, No.1, pp.35-43.
20. Stopar, B., Ambrozic, T., Kuhar, M., and Turk, G., 2006. GPS-derived geoid using artificial neural network and least squares collocation. *Survey Review*, Vol 38, No. 300, pp. 513-524.
21. Vella, M. N. J. P., 2003. Use of similarity transformation to improve GPS heighting. Conf. Proc., Map Asia 2003, Geospatial Media and Communications Pvt. Ltd., Noida, India.
22. Veronez, M. R., Thum, A. B., and Souza, G. C., 2006. A new method for obtaining geoidal undulations through Artificial Neural Networks. In Proceedings of 7 th International Symposium on Spatial Accuracy Assessment in Natural Resources and Environmental Sciences, Lisbon, Portugal.
23. Ziebart, M., Iliffe, J. C., Cross, P. A., Forsberg, R., Strykowski, G., and Tscherning, C. C., 2004. Great Britain's GPS height corrector surface. Proc., ION GNSS 2004, Institute of Navigation, Long Beach, CA, pp.203–210.